GED Math Menagerie!

Steve Schmidt
schmidtsj@appstate.edu
abspd.appstate.edu

Notable Quote

"I've come to a frightening conclusion that I am the decisive element in the classroom. It's my personal approach that creates the climate. It's my daily mood that makes the weather. As a teacher, I possess a tremendous power to make a student's life miserable or joyous. I can be a tool of torture or an instrument of inspiration. I can humiliate or humor, hurt or heal. In all situations, it is my response that decides whether a crisis will be escalated or deescalated and a student humanized or dehumanized."

- Adapted from Haim Ginott

Please Write on this Packet!

You can find everything from this workshop at: abspd.appstate.edu Look under: Teaching Resources and scroll down to GED® Math Menagerie!

This Workshop in One Minute

Today, we will learn research based best practices that will help us to:

- Teach math reasoning through problem solving

- Teach math from concrete to pictorial to abstract

- Understand that students need to be actively involved in learning math

- Teach students to believe they can do math

Some Teaching Points to Consider

- Teach math using a problem solving approach with real world applications

- The GED focuses on reasoning (not computation) and uses real life home and workplace scenarios
- Teach students a problem solving method that works for any type of problem (see the UPS check method on pages 3 - 4.

- Learning is social. Have students work together and talk about what they are doing.

- "The one who does the talking does the learning" Lev Vygotsky
- "The best way to learn something is to teach it" Patricia Wolfe (Brain Matters)

- The instructor acts as a facilitator. This means the instructor:

- works to get students to do as much math as possible
- teaches with questions instead of always telling students what to do
- realizes students' wrong answers are the best opportunity for learning
- avoids telling students they are right/wrong and gets them to think as long as possible

- Our goal should be to produce students who can, without our help, know when and how to use particular strategies for problem solving

- When we tell students every move to make, they develop "learned helplessness."
- What is our end goal: A student who is dependent on us or an independent thinker who can survive and thrive on their own?

> "No matter how kindly, clearly, patiently, or slowly teachers explain, they cannot make students understand. Understanding takes place in the students' minds as they connect new information with previously developed ideas, and teaching through problem solving is a powerful way to promote this kind of thinking."
>
> *Diana Lambdin,* 2003

Sources:
Education Alliance, NIFL
TEAL, US Dept. of Ed

UPS ✔ Problem Solving Method

1. Understand the problem

What are you asked to do?

Will a picture or diagram help you understand the problem?

Can you rewrite the problem in your own words?

2. Create a plan

Use a problem solving strategy:

Guess and check
Make a list
Draw a picture or diagram
Look for a pattern
Make a table
Use a variable
Solve an easier problem
Experiment
Act it out
Work backwards
Change your viewpoint

3. Solve

Be patient

Be persistent

Try different strategies

4. Check

Does your answer make sense?

Are all the questions answered?

What other ways are there to solve this problem?

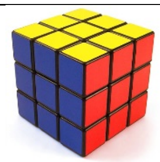
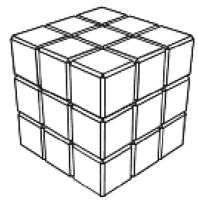
What did you learn from solving this problem?

Source: Polya, How to Solve It

Understand
Plan
Solve
Check

Teach From Concrete to Representational to Abstract

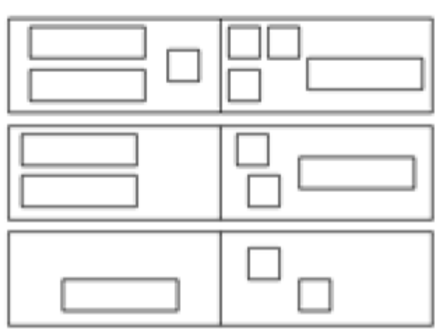
Teach math from concrete to representational to abstract (CRA):

 **Concrete**	 **Representational**	Volume = length x width x height Volume = 3 x 3 x 3 **Abstract**

Starting a math lesson at an abstract level is a common problem. Math expert Mahesh Sharma writes: "In many of the regular classroom teaching situations, the teacher may begin at the abstract form of the concept. As a result, the student may face difficulty in learning the concept or procedure being taught. Even if he has understood the procedure for solving that problem, he may soon forget it.

"Later when the teacher begins a new concept he may assume, incorrectly, that the mastery in the previous concept is still present and therefore may begin the new concept at a higher level, i.e., the abstract level, creating difficulty for the student. This cycle continues and eventually the student begins to lose the teacher's explanations. The student begins to have difficulty in learning mathematics, which then results in failure and that develops a fear of mathematics."

Additionally, remember that many students have failed algebra multiple times before entering our programs. If we try to teach them again using the same abstract methods that did not work before, what makes us think they will work now? The definition of insanity is doing the same thing repeatedly and expecting different results!

Concrete	Representational	Abstract
Student uses algebra tiles to solve the equation. $2x + 1 = 3 + x$ 	Student solves the equation by drawing representations of the concrete model. $2x + 1 = 3 + x$ 	The student connects the concrete models and the pictorial representation to the algebraic methods. $2x + 1 = 3 + x$ $2x + 1 - 1 = 3 + x - 1$ $2x = 2 + x$ $2x - x = 2 + x - x$ $x = 2$

Teaching Functions Using Concrete Examples and Certainty

Often, a function is defined like this: "A function is a relation between a set of inputs and a set of permissible outputs with the property that each input is related to exactly one output." This definition confuses students. Here are some concrete examples of functions:

A Relationship That's Certain and Unique

If we would like students to experience the need for the certainty functions offer us, it's helpful to put students in a place to experience the uncertainty of non-functional relationships first. Put the letters A, B, C, and D on a wall, spaced evenly apart. Ask every student to stand up and use these examples:

Age If you are 20 or under, stand under A If you are 21 to 29, stand under B If you are 30 to 39, stand under C If you are 40 or older, stand under D	**Transportation** If you walked to class today, stand under A If you took the bus to class today, stand under B If you drove to class today, stand under C If you came some other way, stand under D
Duration If it took 10 minutes or less to get to class, stand under A If it took 11 to 20 minutes to get to class, stand under B If it took 21 to 30 minutes to get to class, stand under C If it took more than 31 minutes to get to class, stand under D	
Clothes If you're wearing blue, stand under A. If you're wearing red, stand under B. If you're wearing black, stand under C. If you're wearing white, stand under D.	**Birthday** If you were born in January, stand under A. If you were born in February, stand under B. If you were born in March, stand under C. If you were born in April, stand under D.

See how these last two examples (clothes and birthday) generate a lack of certainty. Students were lulled by the first examples and may now feel a headache.

"I'm wearing white and red, so where do I go?" "I was born in August. There's no place for me to stand."

Now we gather back together and apply formal language to the concepts we've just felt. "Mathematicians call these three relationships 'functions.' Here's why. Why do you think these relationships aren't functions?" Invite students to interrogate the concept of a function in different contexts. Try to keep the focus on certainty – can you predict the output for any input with certainty?

Adapted from Dan Meyer, blog.mrmeyer.com

Function Machine

Use a cardboard box that has holes cut in each end. Label it “The Incredible Function Machine.” Have cards prepared where you have different inputs and outputs. Have students guess if the machine is working properly based on whether each input has exactly one unique output. You can find cards for this activity at: https://abspd.appstate.edu/node/385

The Coke® Machine

An alternative to the function machine but with a similar theme is to use a vending machine like this Coke® machine. Talk about the buttons on the machine and how the first button is Coke®, and the second is Coke Zero® and the third is Sprite® and so on. The buttons are the input and the colas are the outputs.

The idea of one input giving only one output is clear and students also learn quickly that two buttons (inputs) can produce the same colas (output) and that it is still a properly working Coke® machine (function).

We then talk about what it would mean if you pushed the Coke® button and sometimes got a Coke®, sometimes got a Sprite®, and sometimes got a Dasani Water®. This then becomes a reference point for whenever we discuss if a set of numbers is a function or not: “Remember the Coke® machine.”

Adapted from Dan Meyer, blog.mrmeyer.com

Involve Your Students: Teaching with Questions Video

How does this instructor present new material?

How does the instructor make his students curious about new material?

What did he mean by, “Sometimes the best thing I can do is walk away”?

What are the benefits of disagreements in the classroom?

What does the instructor mean when he says, “Wrong answers provide the best opportunity for learning?

Sample Classroom Questions to Help Students Understand Math Reasoning

- What does this mean?
- What are you doing here? (indicating something on student work)
- Tell me where you’re getting each of your numbers from here.
- Why did you decide to…?
- I don’t understand. Could you show me an example of what you mean?
- So what are you going to try next?
- What are you thinking about?
- Is there another idea you might try?
- Why did you decide to begin with…?
- Do you have any ideas about how you might figure out…?
- You just wrote down ____. Tell me how you got that.
- What are you doing there with those numbers?
- Do you agree with ______’s answer? Why or why not?
- Is ______ always true, sometimes true, or never true?

Source: https://youtu.be/hv7MBSEcpW8 Questions adapted from GED Testing Service

How You Can be Good at Math, and Other Surprising Facts about Learning

1. Is there such a thing as a “math brain”?

2. What happens to your brain when you make mistakes in math?

3. What is the growth mindset?

4. What suggestions does Jo Boaler offer for improving math instruction?

5. What else interested you as you watched this video?

Source: Jo Boaler, TED Talk https://www.youtube.com/watch?v=3icoSeGqQtY

Math Bill of Rights

Emotions

I have the right to learn at my own pace and not feel put down or stupid if I'm slower than someone else.

I have the right to feel good about myself regardless of my abilities in math.

I have the right to relax.

I have the right not to base my self-worth on my math skills.

I have the right to dislike math.

Learning Approaches

I have the right to ask whatever questions I have.

I have the right to need extra help.

I have the right to ask a teacher or tutor for help.

I have the right to say I don't understand.

I have the right to not understand.

Self-Worth

I have the right to view myself as capable of learning math.

I have the right to be treated as a competent person.

I have the right to define success in my own terms.

Evaluation Works Both Ways

I have the right to evaluate my math instructors and how they teach.

Source: Sandra L. Davis

GED® Math Content Areas

GED® Math Content Area	Percent of Test Questions
Algebraic Problem Solving	55%
Quantitative Problem Solving	45%

GED® Math Practices

These math practices show the thinking and problem solving skills we need to develop in our students. “The mathematical practices provide specifications for assessing real-world problem-solving skills in a mathematical context rather than requiring students only to memorize, recognize and apply a long list of mathematical algorithms.” (GED® Math Reasoning Assessment Guide)

Math Practices	What Learner Should Know, Understand, and Be Able to Do
MP.1 **Building Solution Pathways and Lines of Reasoning**	- Search for and recognize entry points for solving a problem - Plan a solution pathway or outline a line of reasoning - Select the best solution pathway, according to given criteria - Recognize and identify missing information that is required to solve a problem - Select the appropriate mathematical technique(s) to use in solving a problem or a line of reasoning
MP.2 **Abstracting Problems**	- Represent real world problems algebraically - Represent real world problems visually - Recognize the important and salient attributes of a problem
MP.3 **Furthering Lines of Reasoning**	- Build steps of a line of reasoning or solution pathway, based on previous steps or givens - Complete the lines of reasoning of others - Improve or correct a flawed line of reasoning
MP.4 **Mathematical Fluency**	- Manipulate and solve arithmetic expressions - Transform and solve algebraic expressions - Display data or algebraic expressions graphically
MP.5 **Evaluating Reasoning and Solution Pathways**	- Recognize flaws in others’ reasoning - Recognize and use counterexamples - Identify the information required to evaluate a line of reasoning

GED® Math Most Missed

Quantitative Reasoning

Compute the area and circumference of circles. Find the radius or diameter of a circle when given the area or circumference.
Compute the perimeter and area of polygons. Find side lengths of a polygon when given the perimeter or area.
Compute the perimeter and area of two-dimensional composite shapes, which could include circles.

Algebraic Reasoning

Locate points in the coordinate plane.
Determine the slope of a line from a graph, equation, or table.
Graph two-variable linear equations.

Qualitative Reasoning

Use scale factors to determine the magnitude of size change. Convert between actual drawings and scale drawings.
Solve two step, arithmetic, real world problems involving percents. Examples include but are not limited to: simple interest, tax, markups and markdowns, gratuities and commissions, percent increase and decrease.

Mathematical Practice and Algebraic Reasoning; Recognize Entry Points

Solve one-variable linear equations, and formulas with multiple variables.
Solve linear inequalities in one variable.
Solve one-variable quadratic equations with real solutions, using any appropriate method.

Mathematical Practice and Algebraic Reasoning; Create Algebraic Models

Create linear expressions as part of word-to-symbol translations or to represent situations you have been given.
Create one- or two-variable linear equations to represent situations you have been given.
Create one variable linear inequalities to represent situations you have been given.

Resources

Annenberg Learner

Google: **annenberg learner**
http://www.learner.org/index.html

Has courses of study in such areas as algebra, geometry, and real-world mathematics.

Get the Math

Google: **get the math**
http://www.thirteen.org/get-the-math

Get the Math show how algebra is used in the real world in music, sports, fashion, videogames, restaurants, and special effects.

Manipulatives and Algebra Tiles

Google: **national library of virtual manipulatives** http://nlvm.usu.edu/en/nav/vlibrary.html

This site has algebra manipulatives for instruction across grade levels

Google: **algebra 4 all learnport** http://a4a.learnport.org/page/algebra-tiles

This site provides an easy to use set of virtual algebra tiles.

Real World Algebra

Google: **get the math** http://www.thirteen.org/get-the-math

Get the Math show how algebra is used in the real world in music, sports, fashion, video games, restaurants, and special effects.

Research Base

Dweck, C. S. (2006). *Mindset: The new psychology of success*. New York: Random House.

National Institute for Literacy. (2010). *Algebraic thinking in adult education*. Washington, DC: Author.

Nonesuch, K. (2006). Changing the way we teach math: a manual for teaching basic math to adults. Duncan, B.C., Canada: Malaspina University-College.

Polya, G. (2014). *How to solve it: a new aspect of mathematical method*. Princeton NJ: Princeton Science Library.